B.Sc. Part II (Hons.) Diff. Eqns. 4th Paper

LPDE contd.

1. $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = \sin 3x.$$

Soln. The given equation

$$(D^2 - 2D + 5)y = \sin 3x.$$

For CF

$$D^2 - 2D + 5 = 0$$

$$\Rightarrow D^2 - 2D + 1 = -4 = 4i^2$$

$$\Rightarrow (D-1)^2 = (\pm 2i)^2$$

$$\Rightarrow D = 1 \pm 2i = 1+2i,\ 1-2i$$

$$\therefore CF = A e^{(1+2i)x} + B e^{(1-2i)x}$$

$$= A e^{x} \cdot e^{2ix} + B e^{x} \cdot e^{-2ix}$$

$$= e^{x}\left[A e^{2ix} + B e^{-2ix}\right]$$

$$= e^{x}\left[A(\cos 2x + i\sin 2x) + B(\cos 2x - i\sin 2x)\right]$$

$$= e^{x}\left[(A+B)\cos 2x + i(A-B)\sin 2x\right]$$

$$\Rightarrow CF = e^{x}(C\cos 2x + D\sin 2x)$$

Now, we find PI.

$$PI = \frac{1}{D^2 - 2D + 5} \sin 3x$$

$$\Rightarrow PI = \frac{1}{-3^2 - 2D + 5} \sin 3x \quad \left[\text{Put } D^2 = -3^2\right]$$

$$\Rightarrow PI = \frac{1}{-2D - 4} \sin 3x$$

$$= -\frac{1}{2(D+2)} \sin 3x = -\frac{(D-2)}{2(D^2-4)} \sin 3x$$

$$= \frac{-(D-2)\sin 3x}{2(-3^2-4)}$$

$$= \frac{1}{26}\left[(D-2)\sin 3x\right]$$

$$= \frac{1}{26}\left[D(\sin 3x) - 2\sin 3x\right]$$

$$\Rightarrow PI = \frac{1}{26}\left[3\cos 3x - 2\sin 3x\right]$$

$\therefore$ complete soln is given by

$$y = CF + PI$$

$$\Rightarrow y = e^{x}(C\cos 2x + D\sin 2x) + \frac{1}{26}(3\cos 3x - 2\sin 3x)$$